Rappel

$$T_1 : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$$
$$\vec{x} \longmapsto \begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix}$$

- $\mathrm{Im}(T_1) = \{ T_1(\vec{x}) \mid \vec{x} \in \mathbb{R}^3 \} \subseteq \mathbb{R}^3$

$$T_1\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}x_1\\x_2\\0\end{pmatrix} = x_1\begin{pmatrix}1\\0\\0\end{pmatrix} + x_2\begin{pmatrix}0\\1\\0\end{pmatrix}$$

Donc $\mathrm{Im}(T_1) = \mathrm{span}\left\{ \underbrace{\begin{pmatrix}1\\0\\0\end{pmatrix}}_{\vec{e}_1}, \underbrace{\begin{pmatrix}0\\1\\0\end{pmatrix}}_{\vec{e}_2} \right\} \cancel{=} \mathbb{R}^3$

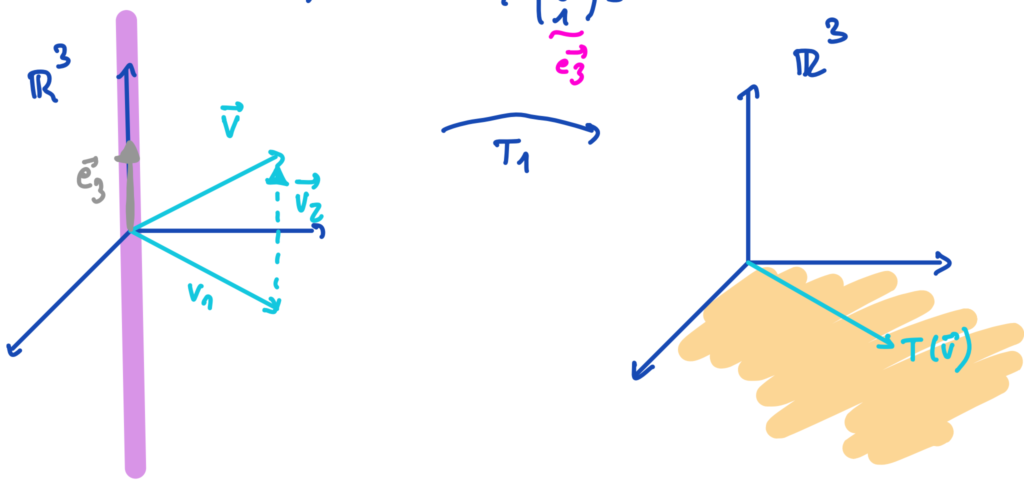
$\subset$ $\neq$

$(\vec{e}_1, \vec{e}_2, \vec{e}_3)$ base canonique de $\mathbb{R}^3$!

- $\mathrm{Ker}(T_1) = \{ \vec{x} \in \mathbb{R}^3 \mid T_1(\vec{x}) = \vec{0} \}$

$$T_1(\vec{x}) = \vec{0} \iff \begin{pmatrix}x_1\\x_2\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

Donc $\mathrm{Ker}(T_1) = \mathrm{span}\left\{ \underbrace{\begin{pmatrix}0\\0\\1\end{pmatrix}}_{\vec{e}_3} \right\}$



On remarque qu'on peut écrire

$\vec{v} = \vec{v}_1 + \vec{v}_2$ avec $\vec{v}_1 \in \mathrm{Im}(T_1)$ et $\vec{v}_2 \in \mathrm{Ker}(T_1)$.